Fault detection problem for discrete-time impulsive system using mixed dissipativity approach

Meng Yao, Guoliang Wei, and Wangyan Li

2024 年 6 月 28 日

Outline

1. Introduction
2. Problem Formulations
3. Mixed Dissipativity Analysis For Fault Diagnosis
   - Mixed dissipativity based system fault sensitivity design
   - Mixed dissipativity based system disturbance insensitivity design
   - Simultaneous mixed fault sensitivity and disturbances insensitivity design
4. Case Studies

Outline

1 Introduction

2 Problem Formulations

3 Mixed Dissipativity Analysis For Fault Diagnosis
- Mixed dissipativity based system fault sensitivity design
- Mixed dissipativity based system disturbance insensitivity design
- Simultaneous mixed fault sensitivity and disturbances insensitivity design

4 Case Studies

Introduction

⏭ the stability issue

📄 Dashkovskiy S, Mironchenko A. "**Input-to-state stability** of nonlinear impulsive systems" *Nonlinear Anal Hybrid Syst.*, vol. 51, no. 3, pp. 1962-1987, 2013.

📄 Liu B, Hill DJ, Sun Z. "Mixed $\mathscr{K}$-dissipativity and stabilization to **ISS** for impulsive hybrid systems" *IEEE Trans Circuits Syst-Exp Briefs.*, vol. 62, no. 8, pp. 791-795, 2015.

⏭ the filtering problem

📄 Pan S, Sun J, Zhao S. "Roust **filtering** for discrete time piecewise impulsive systems" *Signal Process.*, vol. 90, no. 1, pp.324-330, 2010.

📄 Xu J, Sun J. "Finite-time **filtering** for discrete time linear impulsive systems" *Signal Process.*, vol. 92, no. 11, pp. 2718-2722, 2012.

Introduction

the **fault detection** problem

Li W, Yan Y, Bao J. Dissipativity-based distributed fault diagnosis for plantwide chemical process. *J Process Control*. 2020; 96: 37-48.

Zhong M, Xve T, Song Y, Ding SX, Ding EL. Parity space vector machine approach to robust fault detection for linear discrete-time systems. *IEEE Trans Syst Man Cybern Syst*. 2021; 51(7): 4251-4261.

⇓

the results do not consider the **impulsive phenomena**.

⇓

★ deal with **fault detection** issue of linear discrete-time **impulsive systems**.

Method

In this article, we study the **mixed dissipativity** based fault detection problem for a class of **discrete-time impulsive systems**.

Journal of Process Control 96 (2020) 37–48

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Dissipativity-based distributed fault diagnosis for plantwide chemical processes☆

- a novel **mixed fault sensitivity** and **mixed disturbance insensitivity** condition;
- mixed fault sensitivity + mixed disturbance insensitivity $\xrightarrow{\text{mixed supply rate}}$ **mixed dissipativity** condition;
- the novel mixed dissipativity based fault detection approach is developed for discrete-time impulsive systems.

Outline

1 Introduction

2 Problem Formulations

3 Mixed Dissipativity Analysis For Fault Diagnosis
- Mixed dissipativity based system fault sensitivity design
- Mixed dissipativity based system disturbance insensitivity design
- Simultaneous mixed fault sensitivity and disturbances insensitivity design

4 Case Studies

System model

$$\begin{cases} x_{k+1} = Ax_k + Df_k + E\omega_k, & k \neq k_m, \\ x_{k_m+1} = A_{\mathcal{I}}x_{k_m} + D_{\mathcal{I}}f_{k_m} + E_{\mathcal{I}}\omega_{k_m}, & k = k_m, \\ y_k = Cx_k + Wf_k, & k \neq k_m, \\ y_{k_m} = C_{\mathcal{I}}x_{k_m} + W_{\mathcal{I}}f_{k_m}, & k = k_m, \end{cases} \tag{1}$$

- x_k, y_k: state vector, system output;
- f_k: fault signal to be detected;
- ω_k: external disturbance;
- $A, C, D, E, W, A_{\mathcal{I}}, C_{\mathcal{I}}, D_{\mathcal{I}}, E_{\mathcal{I}}, W_{\mathcal{I}}$: known matrices with appropriate dimensions.

$\Rightarrow$ $\mathcal{I}$: impulse

System model

$$\begin{cases} x_{k+1} = Ax_k + Df_k + E\omega_k, & k \neq k_m, \\ x_{k_m+1} = A_{\mathcal{I}}x_{k_m} + D_{\mathcal{I}}f_{k_m} + E_{\mathcal{I}}\omega_{k_m}, & k = k_m, \\ y_k = Cx_k + Wf_k, & k \neq k_m, \\ y_{k_m} = C_{\mathcal{I}}x_{k_m} + W_{\mathcal{I}}f_{k_m}, & k = k_m, \end{cases}$$

- $\{k_m\}_{m\in\mathbb{N}}$: impulsive sequence ($0 = k_0 < k_1 < \cdots < k_m < \cdots$)
- $\tau_m =: k_{m+1} - k_m$: impulsive interval

$$0 < \tau_m < \tau < +\infty$$

where τ is the positive constant which represents the **maximum impulsive interval**.

Aims

Construct a residual-based fault diagnoser such that

1) sensitive to the fault signal (**fault sensitivity**)
2) insensitive to the exogenous disturbance (**disturbance insensitivity**)

1. **fault sensitivity**: the diagnostics unit can quickly and accurately identify fault signals. (**high accuracy**)
2. **disturbance insensitivity**: the diagnostic device can effectively filter out interference signals that are not related to the fault and avoid false alarms. (**high stability**)

Aims

3) fault sensitivity + disturbance insensitivity conditions $\xrightarrow{\text{mixed supply rate}}$ **mixed dissipativity** conditions

- **mixed supply rate**: A function $(\gamma_c(u_c, y_c), \gamma_d(u_d, y_d))$ is called a ***mixed supply rate*** of impulse system if γ_c is locally integrable and γ_d is locally summable

$\Downarrow$

★ **Fault Diagnoser Design**

Fault diagnoser design

- First, a fault diagnoser can be designed as follows:

$$\begin{cases} x_{k+1} = Ax_k + Df_k + E\omega_k, \\ x_{k_m+1} = A_{\mathcal{I}} x_{k_m} + D_{\mathcal{I}} f_{k_m} + E_{\mathcal{I}} \omega_{k_m}, \\ y_k = Cx_k + Wf_k, \\ y_{k_m} = C_{\mathcal{I}} x_{k_m} + W_{\mathcal{I}} f_{k_m}, \end{cases} \Rightarrow \begin{cases} \hat{x}_{k+1} = A\hat{x}_k + L(y_k - \hat{y}_k), \\ \hat{x}_{k_m+1} = A_{\mathcal{I}} \hat{x}_{k_m} + L_{\mathcal{I}} (y_{k_m} - \hat{y}_{k_m}), \\ \hat{y}_k = C\hat{x}_k, \\ \hat{y}_{k_m} = C_{\mathcal{I}} \hat{x}_{k_m}, \end{cases}$$

  - $\hat{y}_k$, $\hat{y}_{k_m}$: the estimates of y_k and y_{k_m};
  - L, $L_{\mathcal{I}}$: the **gains to be designed**.
- Second, the residuals are defined by

$$\begin{cases} r_k = M(y_k - \hat{y}_k), & k \neq k_m, \\ r_{k_m} = M_{\mathcal{I}}(y_k - \hat{y}_k), & k = k_m, \end{cases}$$

  - M, $M_{\mathcal{I}}$: the **residual gain matrices to be designed** as well.

$\Rightarrow$ ★ determine the matrices L, $L_{\mathcal{I}}$, M, $M_{\mathcal{I}}$.

Impulsive error dynamics

Denote estimation error as $e_k = x_k - \hat{x}_k$ and $e_{k_m} = x_{k_m} - \hat{x}_{k_m}$.

$$\begin{cases} x_{k+1} = Ax_k + Df_k + E\omega_k, \\ x_{k_m+1} = A_{\mathcal{I}} x_{k_m} + D_{\mathcal{I}} f_{k_m} + E_{\mathcal{I}} \omega_{k_m}, \\ y_k = Cx_k + Wf_k, \\ y_{k_m} = C_{\mathcal{I}} x_{k_m} + W_{\mathcal{I}} f_{k_m}, \end{cases} \qquad \begin{cases} \hat{x}_{k+1} = A\hat{x}_k + L\left(y_k - \hat{y}_k\right), \\ \hat{x}_{k_m+1} = A_{\mathcal{I}} \hat{x}_{k_m} + L_{\mathcal{I}}\left(y_{k_m} - \hat{y}_{k_m}\right), \\ \hat{y}_k = C\hat{x}_k, \\ \hat{y}_{k_m} = C_{\mathcal{I}} \hat{x}_{k_m}, \end{cases}$$

$$\Rightarrow \begin{cases} e_{k+1} = Ae_k + E\omega_k + Df_k - L\left(y_k - \hat{y}_k\right), & k \neq k_m, \\ e_{k_m+1} = A_{\mathcal{I}} e_{k_m} + E_{\mathcal{I}} \omega_{k_m} + D_{\mathcal{I}} f_{k_m} - L_{\mathcal{I}}\left(y_{k_m} - \hat{y}_{k_m}\right), & k = k_m. \end{cases}$$

Impulsive error dynamics

$$\begin{cases} e_{k+1} = Ae_k + E\omega_k + Df_k - L\left(y_k - \hat{y}_k\right), & k \neq k_m, \\ e_{k_m+1} = A_{\mathcal{I}} e_{k_m} + E_{\mathcal{I}}\omega_{k_m} + D_{\mathcal{I}} f_{k_m} - L_{\mathcal{I}}\left(y_{k_m} - \hat{y}_{k_m}\right), & k = k_m. \end{cases}$$

$$\Rightarrow \begin{cases} e_{k+1} = \tilde{A} e_k + E\omega_k + \tilde{M} f_k, k \neq k_m, \\ e_{k_m+1} = \tilde{A}_{\mathcal{I}} e_{k_m} + E_{\mathcal{I}}\omega_{k_m} + \tilde{M}_{\mathcal{I}} f_{k_m}, k = k_m. \end{cases} \quad (2)$$

$$\tilde{A} = A - LC, \tilde{A}_{\mathcal{I}} = A_{\mathcal{I}} - L_{\mathcal{I}} C_{\mathcal{I}}, \tilde{M} = D - LW, \tilde{M}_{\mathcal{I}} = D_{\mathcal{I}} - L_{\mathcal{I}} W_{\mathcal{I}}.$$

$$\begin{cases} r_k = M(y_k - \hat{y}_k) \\ r_{k_m} = M_{\mathcal{I}}(y_k - \hat{y}_k) \end{cases} \Rightarrow \begin{cases} r_k = MCe_k + MWf_k, & k \neq k_m, \\ r_{k_m} = M_{\mathcal{I}} C_{\mathcal{I}} e_{k_m} + M_{\mathcal{I}} W_{\mathcal{I}} f_{k_m}, & k = k_m. \end{cases}$$

Aims

fault sensitivity+**disturbance insensitivity**$\Rightarrow$? Specific mathematical form

1. Under zero initial condition and $\omega_k = 0$, the effect of fault f_k on the residual r_k and r_{k_m} should be sufficiently large, that is:

$$\sum_{k=k_0+1, k\neq k_m}^{T} \|r_k\|^2 + \sum_{l=0}^{m} \|r_{k_l}\|^2 \geq \beta^2 \sum_{k=k_0+1, k\neq k_m}^{T} \|f_k\|^2 + \beta^2 \sum_{l=0}^{m} \|f_{k_l}\|^2.$$

2. Under zero initial condition and $f_k = 0$, the effect of exogenous disturbance on the residual should be minimized, that is:

$$\sum_{k=k_0+1, k\neq k_m}^{T} \|r_k\|^2 + \sum_{l=0}^{m} \|r_{k_l}\|^2 \leq \gamma^2 \sum_{k=k_0+1, k\neq k_m}^{T} \|\omega_k\|^2 + \gamma^2 \sum_{l=0}^{m} \|\omega_{k_l}\|^2.$$

Threshold

A fault isolation threshold J_{th} will be designed as:

$$J_r = \left(\sum_{s=k-h, s \neq k_s}^{h} r_s^\top r_s + \sum_{k_s \in [k-h, h]} r_{k_s}^\top r_{k_s} \right)^{\frac{1}{2}},$$

$$J_{th} = \sup_{\omega \in l_{2e}, f=0} J_r,$$

$l_2([0, \infty), \mathbb{R}_n)$ is the space of square summable n-dimensional vector-valued functions. The set l_{2e} denotes the extended set of l_2 which consists of the functions whose time truncation lies in l_2.

W. Li, Y. Yan, and J. Bao, "Dissipativity-based distributed fault diagnosis for plantwide chemical processes," *Journal of Process Control*, vol. 96, pp. 37–48, Dec. 2020.

l_2 space

A sequence $\{w_k\}$ belongs to the l_2 space if the sum of its squared elements is finite:

$$\sum_{k=0}^{\infty} |w_k|^2 < \infty$$

$\Rightarrow$ w_k must be bounded and decay over infinite time.

l_{2e} space

A sequence $\{w_k\}$ belongs to the l_{2e} space if it belongs to the l_2 space over some finite time interval. That is, there exists a finite time T such that:

$$\sum_{k=0}^{T} |w_k|^2 < \infty$$

$\Rightarrow$ l_{2e} allows for signals that are energy-limited over finite periods but not necessarily over infinite periods.

In practical systems, external disturbances are often unpredictable, uncertain, and may persist indefinitely. $\Rightarrow$ l_2 ✗ l_{2e} ✓

Diagnosis

Based on the determined threshold J_{th}, the faults can be detected by comparing J_r with J_{th} according to the following rule:

$$\begin{cases} J_r > J_{th} & \Rightarrow \text{with fault} \Rightarrow \text{alarm}, \\ J_r \leq J_{th} & \Rightarrow \text{no faults}. \end{cases}$$

Outline

1. Introduction
2. Problem Formulations
3. Mixed Dissipativity Analysis For Fault Diagnosis
   - Mixed dissipativity based system fault sensitivity design
   - Mixed dissipativity based system disturbance insensitivity design
   - Simultaneous mixed fault sensitivity and disturbances insensitivity design
4. Case Studies

Mixed Dissipativity Analysis For Fault Diagnosis

Mixed dissipative

An impulsive system Σ with input $u_k \in \mathcal{U}_c$, $u_{k_m} \in \mathcal{U}_d$, and output $y_k \in \mathcal{Y}_c$, $y_{k_m} \in \mathcal{Y}_d$ is ***mixed dissipative*** under the mixed supply rate (S_c, S_d), if there exist storage functions $V_c(x_k, k)$ and $V_d(x_k, k)$, $\forall k$ such that

$$\Delta V_c(x_k, k) < S_c(y_k, u_k), \quad k \neq k_m, \quad \forall u_k \in \mathcal{U}_c,$$
$$\Delta V_d(x_k, k) < S_d(y_{k_m}, u_{k_m}), \quad k = k_m, \quad \forall u_{k_m} \in \mathcal{U}_d,$$

where $\Delta V(x_k, k) = V(x_{k+1}, k+1) - V(x_k, k)$.

1. fault sensitive case
   the error dynamics without disturbances is mixed dissipative with the mixed supply rate (S_c, S_d), which defined as $S_c(f_k, r_k) = \|r_k\|^2 - \beta^2\|f_k\|^2$ and $S_d(f_{k_m}, r_{k_m}) = \|r_{k_m}\|^2 - \beta^2\|f_{k_m}\|^2$.
2. disturbance insensitive case
   the mixed supply rates are designed as $S_c(\omega_k, r_k) = \gamma^2\|\omega_k\|^2 - \|r_k\|^2$ and $S_d(\omega_{k_m}, r_{k_m}) = \gamma^2\|\omega_{k_m}\|^2 - \|r_{k_m}\|^2$

- ? There is no u in the model.

$$\text{mix dissipative}\begin{cases}\Delta V_c(x_k,k) < S_c(y_k,u_k),\\ \Delta V_d(x_k,k) < S_d(y_{k_m},u_{k_m}),\end{cases}$$

- ? Why not y_k and u_k.

$$S_c(f_k,r_k) = \|r_k\|^2 - \beta^2\|f_k\|^2$$
$$S_d(f_{k_m},r_{k_m}) = \|r_{k_m}\|^2 - \beta^2\|f_{k_m}\|^2$$

Note: u_k represents exogenous inputs. Here, it represents either faults f_k or disturbances ω_k.

Mixed dissipativity based system fault sensitivity design

Theorem 1

The error dynamics (2) without disturbances is ***mixed dissipative*** under the mixed supply rate (S_c, S_d) with $S_c(f_k, r_k) = \|r_k\|^2 - \beta^2\|f_k\|^2$ and $S_d(f_{k_m}, r_{k_m}) = \|r_{k_m}\|^2 - \beta^2\|f_{k_m}\|^2$, if there exist matrices Y, $Y_{\mathcal{I}}$, positive semi-definite matrices Z, $Z_{\mathcal{I}}$, a positive definite matrix P, scalar β such that the following LMIs hold:

$$\Pi_1 = \begin{bmatrix} P & PA - YC & PD - YW \\ & P + C^\top ZC & C^\top ZW \\ & * & W^\top ZW - \beta^2 I \end{bmatrix} > 0, \tag{3}$$

$$\Pi_2 = \begin{bmatrix} P & PA_{\mathcal{I}} - Y_{\mathcal{I}}C_{\mathcal{I}} & PD_{\mathcal{I}} - Y_{\mathcal{I}}W_{\mathcal{I}} \\ & P + C_{\mathcal{I}}^\top Z_{\mathcal{I}}C_{\mathcal{I}} & C_{\mathcal{I}}^\top Z_{\mathcal{I}}W_{\mathcal{I}} \\ & * & W_{\mathcal{I}}^\top Z_{\mathcal{I}}W_{\mathcal{I}} - \beta^2 I \end{bmatrix} > 0. \tag{4}$$

Mixed dissipativity based system fault sensitivity design

In this case, the gains of the desired diagnoser can be obtained as follows:

$$L = P^{-1} Y, \quad L_{\mathcal{I}} = P^{-1} Y_{\mathcal{I}}, \quad Z = M^{\top} M, \quad Z_{\mathcal{I}} = M_{\mathcal{I}}^{\top} M_{\mathcal{I}}$$

The residual gains M and $M_{\mathcal{I}}$ can be obtained by factorizing $Z, Z_{\mathcal{I}}$.

证明.

- **Part 1.** Mixed dissipative.
- **Part 2.** Mixed fault sensitivity.

Omitted here. □

Mixed dissipativity based system disturbance insensitivity design

Theorem 2

The impulsive error dynamics (2) without faults is ***mixed dissipative*** under the mixed supply rate (S_c, S_d) with $S_c(\omega_k, r_k) = \gamma^2\|\omega_k\|^2 - \|r_k\|^2$ and $S_d(\omega_{k_m}, r_{k_m}) = \gamma^2\|\omega_{k_m}\|^2 - \|r_{k_m}\|^2$, if there exist matrices Y, $Y_{\mathcal{I}}$, positive semi-definite matrices Z, $Z_{\mathcal{I}}$, positive definite matrix P, scalar γ such that the following LMIs hold:

$$\Gamma_1 = \begin{bmatrix} P & PA - YC & PE \\ & P - C^\top ZC & 0 \\ & * & \gamma^2 I \end{bmatrix} > 0, \quad (5)$$

$$\Gamma_2 = \begin{bmatrix} P & PA_{\mathcal{I}} - Y_{\mathcal{I}}C_{\mathcal{I}} & PE_{\mathcal{I}} \\ & P - C_{\mathcal{I}}^\top Z_{\mathcal{I}}C_{\mathcal{I}} & 0 \\ & * & \gamma^2 I \end{bmatrix} > 0. \quad (6)$$

Mixed dissipativity based system disturbance insensitivity design

In this case, the gains of the desired fault diagnostic observer can be obtained as follows:

$$L = P^{-1}Y, \quad L_{\mathcal{I}} = P^{-1}Y_{\mathcal{I}}, \quad Z = M^{\top}M, \quad Z_{\mathcal{I}} = M_{\mathcal{I}}{}^{\top}M_{\mathcal{I}}$$

The residual gains M and $M_{\mathcal{I}}$ can be obtained by factorizing Z, $Z_{\mathcal{I}}$.

证明.

- **Part 1.** Mixed dissipative.
- **Part 2.** Mixed disturbances insensitivity.

Omitted here. □

Simultaneous mixed fault sensitivity and disturbances insensitivity design

Theorem 3

The impulsive error dynamics (2) satisfies the fault sensitivity, disturbances insensitivity conditions simultaneously, if there exist matrices Y, $Y_{\mathcal{I}}$, positive semi-definite matrices Z, $Z_{\mathcal{I}}$, a positive definite matrix P, and scalars β, γ such that the following LMIs hold:

$$\begin{aligned} \max_{\beta,\gamma} \quad & \beta - \gamma \\ \text{s.t.} \quad & \Pi_1 > 0, \;\; \Pi_2 > 0 \;\; \Gamma_1 > 0, \;\; \Gamma_2 > 0. \end{aligned} \tag{7}$$

Simultaneous mixed fault sensitivity and disturbances insensitivity design

In this case, the gains of the desired fault diagnostic observer can be obtained as follows:

$$L = P^{-1}Y, \quad L_{\mathcal{I}} = P^{-1}Y_{\mathcal{I}}, \quad Z = M^{\top}M, \quad Z_{\mathcal{I}} = M_{\mathcal{I}}^{\top}M_{\mathcal{I}}$$

The residual gains M and $M_{\mathcal{I}}$ can be obtained by factorizing Z, $Z_{\mathcal{I}}$.

证明.

The proof is straightforward with the combination of Theorems 1 and 2. □

Outline

1. Introduction
2. Problem Formulations
3. Mixed Dissipativity Analysis For Fault Diagnosis
   - Mixed dissipativity based system fault sensitivity design
   - Mixed dissipativity based system disturbance insensitivity design
   - Simultaneous mixed fault sensitivity and disturbances insensitivity design
4. Case Studies

Parameters

$$
\begin{cases}
x_{k+1} = Ax_k + Df_k + E\omega_k, & k \neq k_m, \\
x_{k_m+1} = A_{\mathcal{I}}x_{k_m} + D_{\mathcal{I}}f_{k_m} + E_{\mathcal{I}}\omega_{k_m}, & k = k_m, \\
y_k = Cx_k + Wf_k, & k \neq k_m, \\
y_{k_m} = C_{\mathcal{I}}x_{k_m} + W_{\mathcal{I}}f_{k_m}, & k = k_m,
\end{cases}
$$

- coefficient matrix

$$
A = \begin{bmatrix} 0.3658 & 0.16 \\ 0.6532 & 0.6169 \end{bmatrix}, A_{\mathcal{I}} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, E_{\mathcal{I}} = \begin{bmatrix} -0.1 \\ 0.7 \end{bmatrix}, W = 1,
$$

$$
D = \begin{bmatrix} 0.8 \\ -1 \end{bmatrix}, D_{\mathcal{I}} = \begin{bmatrix} 0.7 \\ -1 \end{bmatrix}, C = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, C_{\mathcal{I}} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, E = \begin{bmatrix} -0.1 \\ 0.7 \end{bmatrix}, W_{\mathcal{I}} = 1.
$$

Gains

- impulse sequence、interval

$$k_m = 5l \ (l = 0, 1, 2, \cdots) \Rightarrow \tau_m = 5$$

- Calculate the LMIs (3), (4) and (5), (6) obtained in Theorems 1 and 2, respectively.

Table 1: Observer gains and other calculated variables

Variables	β	γ	M	$M_{\mathcal{I}}$	P	L	$L_{\mathcal{I}}$
Values	2.225	0.7136	4.5115	3.9875	$\begin{bmatrix} 1.4036 & 0.2736 \\ 0.2736 & 0.3453 \end{bmatrix}$	$\begin{bmatrix} 0.3768 \\ 2.843 \end{bmatrix}$	$\begin{bmatrix} 0.3854 \\ 2.7583 \end{bmatrix}$

Noise/Fault

- The system external noise

$$\omega_k = 0.4\sin(\frac{\pi}{4}k).$$

- The fault signal

$$f(x) = \begin{cases} 0.1, & 20 \leq k \leq 30, \\ 0, & \text{otherwise}. \end{cases}$$

Evaluation of $J(k)$

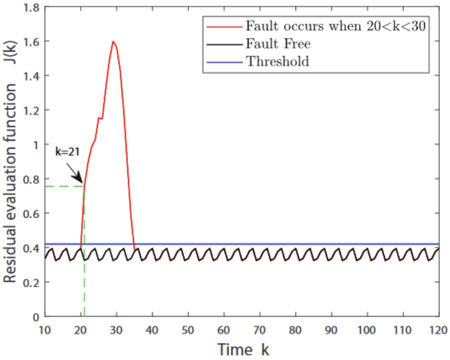


Figure 1: Residual evaluation function $J(k)$

Mass-spring system

$$m_1\ddot{q}_1(t) + (k_1 + k_2)q_1(t) - k_2 q_2(t) = F,$$
$$m_2\ddot{q}_2(t) - k_2 q_1(t) + k_2 q_2(t) = 0,$$

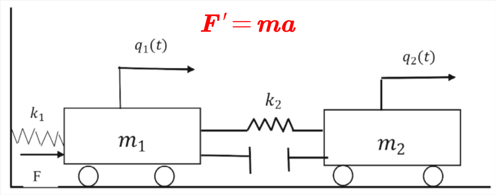
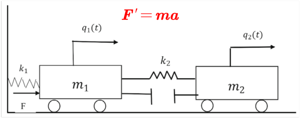
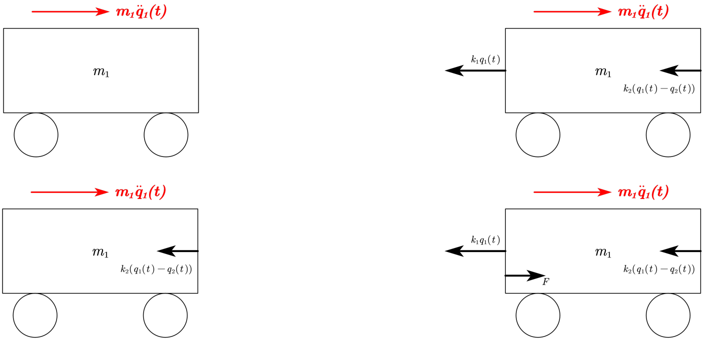


Figure 2: Two-mass systems with constraint buffer

Mass-spring system





$$\Rightarrow m_1\ddot{q}_1(t) = -k_1 q_1(t) - k_2(q_1(t) - q_2(t)) + F$$

Parameters

$$m_1\ddot{q}_1(t) + (k_1 + k_2)q_1(t) - k_2 q_2(t) = F,$$
$$m_2\ddot{q}_2(t) - k_2 q_1(t) + k_2 q_2(t) = 0,$$

- coefficient

$$m_1 = 1.5\text{kg},\ m_2 = 0.5\text{kg},\ k_1 = 1\text{N/m},\ k_2 = 0.5\text{N/m}.$$

- F is the external force and represents the **fault signal** in this mass-spring system.

$$F = \begin{cases} 0.088\text{N}, & 50 \le k \le 55, \\ 0, & \text{otherwise.} \end{cases}$$

Parameters

- the loss of kinetic energy due to a collision

$$m_1\dot{q}_1(t_m+1)+m_2\dot{q}_2(t_m+1)=m_1\dot{q}_1(t_m)+m_2\dot{q}_2(t_m),$$
$$\dot{q}_1(t_m+1)-\dot{q}_2(t_m+1)=-\delta(\dot{q}_1(t_m)-\dot{q}_2(t_m)),\qquad (8)$$

  where $\delta\in[0,1)$, is the coefficient restitution.

Haddad WM, Chellaboina V, Nersesov SG. Impulsive and Hybrid Dynamical Systems: Stability, Dissipativity, and Control. *Princeton University Press*; 2006.

- $x(t)=[q_1(t),\dot{q}_1(t),q_2(t),\dot{q}_2(t)]^\top$
  the collision occurs when $x\in\mathcal{J}=\{x_1-x_3=l\}$
  - x_i: the i-th component of the state $x(t)$
  - l: the total buffer length
- $y=[x_1^\top,x_3^\top]^\top$

Parameters

$$\Rightarrow \begin{cases} \dot{x}(t) = Ax(t) + DF, & k \neq k_m, \\ x_{k_m+1} = A_{\mathcal{I}} x_{k_m}, & k = k_m, \end{cases} \tag{9}$$

where

$$A_{\mathcal{I}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{(1+\delta)m_2}{m_1+m_2} & 0 & \frac{(1+\delta)m_2}{m_1+m_2} \\ 0 & 0 & 1 & 0 \\ \frac{(1+\delta)m_1}{m_1+m_2} & 0 & -\frac{(1+\delta)m_1}{m_1+m_2} & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}.$$

Parameters

- Let the sampling interval $S = 0.5s$

$$\begin{cases} \dot{x}(t) = Ax(t) + DF, & k \neq k_m, \\ x_{k_m+1} = A_{\mathcal{I}} x_{k_m}, & k = k_m, \end{cases} \Rightarrow \begin{cases} x_{k+1} = \bar{A} x_k + H f_k, & k \neq k_m, \\ x_{k_m+1} = A_{\mathcal{I}} x_{k_m}, & k = k_m, \end{cases} \quad (10)$$

where $\bar{A} = e^{AS}$, $H = \left(\int_0^S e^{At} dt \right) D$.

- Noise

$$\Rightarrow \begin{cases} x_{k+1} = \bar{A} x_k + H f_k + E\omega_k, & k \neq k_m, \\ x_{k_m+1} = A_{\mathcal{I}} x_{k_m}, & k = k_m, \end{cases} \quad (11)$$

where $\omega_k = \sin\left(\frac{\pi}{4} k\right)$

Gains

By solving the LMI (7), the feasible solutions are obtained in Table 2.

Table 2: Observer gains and other calculated variables

Variables	β	γ	M	L
Values	0.7985	0.5698	1.146	$\begin{bmatrix} -0.0938 \\ -0.4743 \\ 0.8897 \\ 1.6210 \end{bmatrix}$

Evaluation of $J(k)$

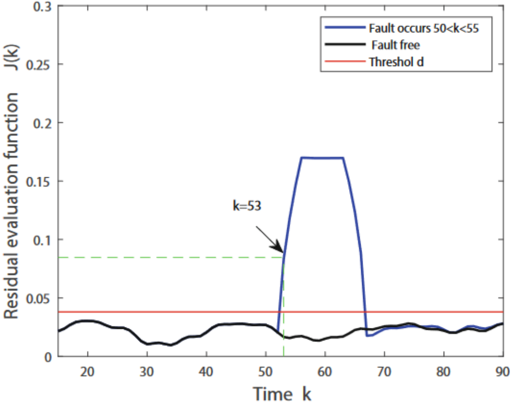


Figure 3: Residual evaluation function $J(k)$

Thank you for your attention!